Mode Coupling for Phonons in a Single-Layer Dusty Plasma Crystal

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New modes in a dusty plasma result from coupling of differently polarized phonons. A single horizontal layer of charged microparticles, confined so that vertical as well as horizontal motions are possible, usually exhibits three modes. An experiment shows that mode coupling leads to a new hybrid mode and another new mode. Coupling also leads to a recently reported hybrid mode and nondispersive mode, shown here to occur in an unmelted lattice. A linear theory based on ion wakes is able to predict some, but not all, of these modes. Other multiphase systems could exhibit similar mode coupling.

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Dusty plasma consists of charged microparticles dispersed in an ionized gas. As in other multiphase systems such as colloids, the microparticles in dusty plasma can self-organize. Because they have a large charge and are confined, self-organization can result in a crystalline structure [1,2], which exhibits phonons [3,4]. Here we expand what was previously known about “mode coupling,” which is a modification of the phonon modes in a dispersed phase, enhanced by a flow in a continuous phase.

Observing this mode-coupling phenomenon requires an experimental system with a flow (directed energy) in a continuous phase, and a dispersed phase that sustains multiple modes. Here, the term “mode” refers to an oscillation, or phonon. Random particle motions, for example, due to thermal fluctuations, can be decomposed into these modes. If collective effects are significant, the frequency $\omega$ of a mode usually varies with wave number $k$, but if $\omega$ is independent of $k$, we will term the mode “nondispersive.” Modes are distinguished by the direction of particle velocity with respect to wave propagation, which we term “polarization.” In a crystalline lattice, there are longitudinal acoustic (LA) and transverse acoustic (TA) modes. If the experimental system has a suspension of dispersed microparticles that fill only a single layer, it will also have a third mode: an out-of-plane transverse optical mode (TO). The TO mode is dominated by oscillatory motion in the vertical confining potential, which is characterized by a resonance frequency, $\omega_{\text{res}}$ [5]. (The wave number $k$ is in-plane for all three modes.) These three modes have been previously studied in simulations with a Yukawa interparticle potential and a parabolic confining potential in the out-of-plane $z$ direction, for example [6–8].

An experimental system that meets these requirements is a single-layer suspension of microparticles levitated in a plasma, i.e., dusty plasma. Its microparticles move slowly enough, and are large enough, that videomicrography can be used to track their motion. In our experiment, the microparticles comprise a strongly coupled plasma, and they self-organize in a crystalline lattice. Ions in the plasma represent a continuous phase that flows past the microparticles due to an electric field.

Recently, in what was perhaps the first experimental demonstration of mode coupling for a multiphase system with a directed flow of a continuous phase, Couédel et al. [9] observed coupling of the LA and TO modes in a single-layer dusty plasma. The dispersion relations of the modes were greatly modified, especially with the appearance of the LA-TO hybrid mode. A hybrid mode can occur when the dispersion relations for two different modes with different polarizations happen to have the same frequency $\omega$ and wave number $k$. The corresponding particle motion has a mixture of two directions. In the experiment of Ref. [9], a suspension of charged microspheres was electrically confined in a single layer, and there was an ion flow perpendicular to the layer. In addition to the LA-TO hybrid mode, what we term the nondispersive (ND) mode also appeared in their reported spectra. Their analysis was largely centered on large-amplitude particle motion in an extreme case that leads to melting of the crystalline structure of the single layer.

Here, we will report observations of two additional modes in an experiment with a crystalline structure. These two modes have apparently not been reported or predicted previously: a longitudinal mode that we term the LA-TO hybrid and ND modes which we term the TA-ND hybrid mode. In addition to these two main results, we also find that the LA-TO hybrid and ND modes observed recently in an unstable melted lattice [9] can also occur in our stable crystalline lattice.

A frictionless linear theory predicts that mode coupling can excite hybrid modes in a single-layer dusty plasma suspension [10–12]. Mode coupling can be greatly enhanced if a system is nonequilibrium, for example, when an external power supply drives an ion flow in a dusty plasma, leading to an ion wake downstream of each charged microsphere [13–16]. An ion wake has a localized accumulation of positive charge.

In the frictionless linear theory [10–12], the effects of ions and electrons are replaced by a screening length $\lambda_p$ and a hypothetical positively charged point particle, which...
is rigidly attached a distance $d_i$ downstream of a negatively charged microsphere. Phonon modes are identified by an eigenvalue analysis of a dynamical matrix. (This matrix is a Fourier transform of a force constant matrix obtained by linearizing potentials between a micro-particle and numerous neighbors and point particles, ignoring gas friction.) For a given wave number $k$, there are two eigenvalues $\omega^2$, which can be identical. If they are complex, the mode is either unstable or damped. A dispersion relation is found by plotting $\omega$ vs $k$. Eigenvectors indicate velocity direction (polarization).

To illustrate the role of ion wakes, we sketch in Fig. 1 a simple calculation showing that a small vertical displacement of a microparticle in a single layer results in a horizontal net force applied to neighboring particles. The horizontal force is weaker (second order in the displacement) in the absence of a wake effect, but stronger (first order) with the wake effect. Thus, the coupling of horizontal and vertical motions associated with low amplitude waves, like those reported here, is most likely to be observed when there is a wake potential.

In the experiment, we used two high-resolution video cameras to measure both in-plane and out-of-plane motions precisely and simultaneously. A top-view camera, with a field of view (FOV) of $37 \times 23$ mm$^2$, imaged the single layer in a horizontal $x$-$y$ plane. A side-view camera, with a FOV of $18 \times 10$ mm$^2$, imaged a cross section in the vertical $x$-$z$ plane. Images of 1152 $\times$ 720 pixel$^2$ were recorded at 10 ms intervals for 50 s.

Our multiphase system consisted of 8.09 $\mu$m melamine formaldehyde polymer microparticles introduced into an rf plasma. A dc self-bias of $-57$ V drove a steady ion flow toward a horizontal lower electrode. More than 15,000 microparticles were levitated in a single horizontal layer 8 mm above the electrode. (With only one layer, our system is not expected to exhibit modes peculiar to bilayer systems.) Microparticles self-organized in a crystallinelike lattice, Figs. 2(a) and 2(b). An additional phase was neutral argon gas at 20 mtorr, which interacts with the microparticles by exerting a frictional force. The damping rate, $3.5$ s$^{-1}$, was low enough that the modes reported here were not overdamped.

For this experiment, it is important that the suspension be a nearly flat horizontal layer, without significant curvature. If it were curved, an out-of-plane velocity would have a horizontal vector component that our cameras would misidentify as being in-plane. In a test, we found that our suspension was planar to less than 0.03 rad, so that the fractional error in the energy in the spectrum will be negligible, $< (0.03)^2$.

We performed a main experiment that had stronger interparticle interactions ($\kappa = b/\lambda_D = 0.9$) and a control experiment with less strong interactions ($\kappa = 1.4$). This was accomplished by introducing more particles into the plasma to achieve a smaller particle spacing for the main experiment. The lattice constant was $b = 0.6$ mm and 1.0 mm for the main and control experiments, respectively. Both experiments had the same conditions for gas, ions, electrons, screening length, and confining potentials. In both experiments, the collection of microparticles comprised a strongly coupled plasma.

Our results are measurements of phonon spectra [5] for three polarizations, corresponding to random particle motion in the in-plane $\hat{x}$ and $\hat{y}$ directions, and the out-of-plane $\hat{z}$ direction. The analysis begins with measuring microparticle positions and velocities in each video frame [17]. Choosing a wave number $k$ in the in-plane $\hat{x}$ direction, we compute the longitudinal particle current $j_x(k, t) = \sum v_{x,i}(t) e^{-i k x_i(t)}$ using position $x_i$ and velocity $v_{x,i}$ data from the top-view camera. Similarly, we find the transverse particle currents, $j_y(k, t) = \sum v_{y,i}(t) e^{-i k y_i(t)}$ and $j_z = \sum v_{z,i}(t) e^{-i k z_i(t)}$, using $x_i$ and $v_{y,i}$ from the top-view camera, or $x_i$ and $v_{z,i}$ from the side-view camera. The phonon spectrum (a graph of energy as a function of $k$ and $\omega$) is

![FIG. 1](color online). Vectors for forces acting on microparticles, calculated for two simple cases. (a) No ion-wake effect: microparticles, with negative charge $Q$, interact only with nearest neighbors via a Yukawa potential. (b) With ion-wake effect: microparticles also interact with a point particle with positive charge $q_i$ located a distance $d_i$ below the neighboring microparticles. In these calculations, microparticles are assumed to be disturbed vertically, but not horizontally, with a sinusoidal amplitude. Parameters assumed were $\Delta/\lambda_D = 1.0$, $q_i = -0.5Q$, and $d_i = 0.5\Delta$.

![FIG. 2](color online). (a) A portion of an image from the top-view camera, which was used to compute (b) the static structure factor [19]. Both show a crystalline structure. (c) Image from the side-view camera. These images are from the main experiment, with a lattice constant $b = 0.6$ mm. The control experiment was similar, but with $b = 1.0$ mm.
tions of the LA and TO modes, i.e., at the dispersion relation in Ref. [9], and the others are new.

The two modes seen in our main experiment that were recently observed by Couédel et al. [9] are indicated in italics in Figs. 4(a)–4(c); the first is the LA-TO hybrid mode, for the polarization with microparticle motion in the $x$ direction. This new hybrid mode, in Fig. 4(b), has the same polarization as the typical LA mode, but a much higher frequency at small wave numbers. This new LO mode has negative dispersion, i.e., $\omega$ diminishes with $k$, and its dispersion relation coincides with that of the TO mode. Because the latter mode has motion in the $\hat{z}$ direction, this LO mode appears to be the result of horizontal motion coupling with the vertical motion associated with the TO mode. This coupling is linear, because nonlinear coupling would likely result in a frequency or wave number that is different from the existing TO mode. (We verified that the energy of our LO mode is too high for the LO mode to be explained merely as a misidentification of the TO mode due to suspension curvature.)

The second new mode we observed is an in-plane transverse hybrid mode, for the polarization with microparticle motion in the $y$ direction. This new hybrid mode, in Fig. 4(c), appears at the intersection of the TA and nondispersive (ND) modes. Thus, we interpret it as a hybrid of the TA and ND modes. This new TA-ND hybrid mode has an unusually large portion of kinetic energy for in-plane motion, Fig. 4(f), but not out-of-plane motion, Fig. 4(d).

We now compare the $\omega$ vs $k$ dispersion relations of the two new modes with the smooth curves in Figs. 4(a)–4(c) predicted by the frictionless linear theory of Ref. [12]. We find that the new LO mode, but not the new TA-ND hybrid mode, are branches of the dispersion relation predicted by the theory [12]. We also find that the eigenvectors indicate a mixture of $v_x$ and $v_y$ motion for the LO mode (and for the LA-TO hybrid mode, as well).
This comparison indicates that all the required physics for the LO and LA-TO modes are present in the frictionless linear theory, but not for the TA-ND hybrid mode. Some physics, such as nonlinearity, must be lacking from the model because it fails to predict the TA-ND hybrid mode, the ND mode itself, and its harmonic. Our results also demonstrate that hybrid modes resulting from mode coupling do not always cause melting. They also occur in our crystal, which was observed to be stable. This result should be compared to previous experiments [9,18], which indicated that high-amplitude motion associated with hybrid modes can melt a lattice. We conclude that melting is not an inevitable consequence of mode coupling.

In summary, we observed new modes: the LO mode which results from mode coupling, and the TA-ND hybrid mode which occurs at the intersection of the dispersion relations of the TA and ND modes. We confirmed that the previously observed ND and LA-TO hybrid modes can occur in stable crystalline lattices, without melting. A simple frictionless linear theory based on the ion-wake effect [12] cannot explain the ND and TA-ND modes.

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