Pressure and energy of compressional shocks in two-dimensional Yukawa systems

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The propagation of compressional shocks in two-dimensional (2D) dusty plasmas is investigated using MD simulations under various conditions. The shock Hugoniot curves of the relationship between the shock front speed $D$ and the mean particle speed $v_f$ after shocks are obtained and analytically fit to parabolic expressions. As the screening parameter increases, the weaker Yukawa interparticle interaction causes the shock Hugoniot curves to be more linear. Combining the obtained shock Hugoniot curves with the Rankine-Hugoniot jump relations, analytic expressions of pressure and energy after the shocks in 2D Yukawa systems are obtained, which are functions of the observable quantities, like the shock front speed $D$ or the mean particle speed $v_f$ or the specific volume.

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I. INTRODUCTION

A shock wave, moving faster than the local sound speed of medium, is usually described as a propagating disturbance, where extremely rapid rises of pressure, temperature, and density occur [1]. The understanding of shock waves is important for a wide range of applications, such as the measurements of equations of state (EOS) [2,3], high explosives [4], and particle acceleration [5–7]. As one of the key physical quantities, the internal pressure plays an important role in the investigation of shocks [2,3].

After applying a compression to a system with a high speed, a shock would be generated [8,9], with a shock front propagating faster than the sound speed. However, during this compression, the variation of physical quantities, such as the internal pressure or energy, is very complicated. In typical physical systems, even in equilibrium, it is difficult to directly measure the internal pressure and energy, unlike observable quantities, such as the shock front speed $D$. Theoretically, the pressure in an equilibrium system can be calculated from the stress tensor, as performed in a two-dimensional (2D) Yukawa liquid in Ref. [10]. Here, we seek a method to obtain the physical quantities of the internal pressure and energy of shocks, from easily observable quantities, such as the shock front speed.

As an excellent model system, dusty plasma [11–15] is a four-component mixture, consisting of free electrons, ions, gas atoms, and electrically charged micron-sized dust particles. Because of their high charge $Q$ and low charge-to-mass ratio, these dust particles are strongly coupled [16]. In typical experiments, one could choose monodispersed dust particles to easily form a 2D suspension of dust particles [17]. Due to the shielding effects of free electrons and ions, the interaction between dust particles can be modeled as the Yukawa potential [18,19]. In 2D dusty plasmas, the motion of dust particles can be directly video imaged by high-speed cameras, so that the dynamics of individual particles can be studied at the kinetic level [20]. The coupling parameter $\Gamma = Q^2/(4\pi\epsilon_0k_B\bar{T})$ and the screening parameter $\kappa = a/\lambda_D$ are often used to characterize 2D dusty plasmas, or 2D Yukawa systems. For 2D systems, $a = (\pi n)^{-1/2}$ is the Wigner-Seitz radius for an areal number density of $n$, and $T$ is the kinetic temperature of dust particles.

A few shock experiments have already been carried out in dusty plasmas [21–25]. In Ref. [21], two shock modes are found, one is the fast mode corresponding to an acoustic wave, the other is the slow mode corresponding to a dust-acoustic wave. In Ref. [22], it is found that the shock melting occurs in two stages: first the lattice is compressed in the direction of shock propagation, then after the shocks the particle velocities are randomly scattered in all directions. In Ref. [24], a bow shock structure is observed in a 2D supersonic dust plasma flow, suggesting a unique nature of the hydrodynamic non-isothermal process of dust plasma. Besides these experiments, using particle-level simulations of shocks, the thermodynamic and kinetic properties of shocks in 2D dusty plasmas have been reported in Ref. [9]. Although the pressure and energy of 2D dusty plasmas (or 2D Yukawa liquids) have been systematically studied in equilibrium [10,26–28], in our literature search, we have not found the investigation of these physical quantities during shocks in dusty plasmas or Yukawa systems. In this paper, we look for the variations of these physical quantities, like the pressure and energy, after the shocks in 2D dusty plasmas or Yukawa systems, expressed as functions of the observable quantities, like the shock front speed $D$.

Here, we use MD simulation of Yukawa systems to mimic compressional shocks in 2D dust plasmas. A piston, modeled by a reflecting boundary condition, moving at a constant speed $v_p$ is used to generate shocks. In Sec. II, we describe our simulation method. In Sec. III, we first calculate the shock front speed $D$ and the mean particle speed $v_f$ after
shocks, which both can be accurately measured in dusty plasma experiments. The relationship between $D$ and $\bar{v}$ is called the shock Hugoniot curve. Then, combining the shock Hugoniot curve with the Rankine-Hugoniot jump relations [1] for the conservations of mass, momentum, and energy, we obtain analytic expressions of pressure and energy after the shocks in 2D Yukawa systems. Note that, we mainly focus on the thermodynamic property of the postshock region (after shocks).

II. SIMULATION METHOD

We perform frictionless MD simulations of 2D Yukawa systems to mimic the propagation of the shocks in 2D dusty plasmas using LAMMPS [9,29]. In our simulation, the equation of motion for each dust particle $i$ is

$$m \ddot{\mathbf{r}}_i = -\nabla \sum_{j \neq i} N \phi_{ij} + F_{i}^{\text{pis}} + F_{i}^{\text{conf}},$$  \hspace{1cm} (1)

where the particle-particle interaction $\phi_{ij}$ is chosen as the Yukawa repulsion of

$$\phi_{ij} = Q^2 \exp(-r_{ij}/\lambda_D)/4\pi \varepsilon_0 r_{ij}.$$  \hspace{1cm} (2)

Here, $Q$ is the charge of each dust particle, and $\lambda_D$ is the Debye screening length. On the right-hand side of Eq. (1), the two terms of $F_{i}^{\text{pis}}$ and $F_{i}^{\text{conf}}$ refer to the forces from the moving piston and the fixed confinement, respectively, as we explain shortly.

As shown in Fig. 1, shocks would substantially modify the Yukawa systems. For example, in the preshock region the screening parameter $\kappa$ is larger than that in the postshock region. Here, the preshock and postshock mean the undistributed and the steady planar shock regions, respectively, while hard collisions happen at the shock front (between the preshock and postshock regions) [9]. To distinguish the parameters in these regions, here we use the symbols with the subscript 0 to represent the physical quantities of the preshock region, like $k_0$ and $\Gamma_0$, while the symbols without the subscript 0 as the physical quantities in the postshock region.

Here are our simulation methods to mimic the moving piston and the fixed confinement. We simulate $N = 16384$ dust particles constrained within a 2D plane with the dimensions of $485.4 a_0 \times 105.6 a_0$. In the equation of motion, Eq. (1), $F_{i}^{\text{pis}}$ is used to mimic the force from the piston, which moves from left to right at a constant speed of $v_p$ to generate shocks, as shown in Fig. 1. Then, the generated shock front would propagate at another constant speed, marked as $D$, in the undistributed preshock region. We use the form of a Gaussian function $F_{i}^{\text{pis}} = 50 \exp[-(x - x_p)^2/0.25a_0^2] m_0 a_0^2 \kappa$ to mimic the force from this piston, which has the maximum at the location of the piston $x_p$. Here, $\omega_{pd} = (Q^2/(2\pi \varepsilon_0 m a_0^3))^{1/2}$ is the nominal dusty plasma frequency. As in Ref. [9], the force $F_{i}^{\text{pis}}$ is specified in the $x$ direction, as the reflecting boundary. In our MD simulations, we use the periodic boundary in the $y$ direction, as in Ref. [9]. However, in the $x$ direction, i.e., the moving direction of the piston, the periodic boundary cannot be used any more. Instead, at the right edge boundary, we use the inward Gaussian distribution force in the $x$ direction $F_{i}^{\text{conf}} = 50 \exp[-(x - 242.7 a_0)^2/0.25a_0^2] m_0 a_0^2 \kappa$ (the same magnitude as the moving piston) to confine these simulated particles. We justify that this chosen force $F_{i}^{\text{conf}}$ is big enough to confine these particles within the simulation box, with our simulation conditions.

Here we list our specified parameters in our MD simulations. Before moving the piston, we specify the coupling parameter as the constant of $\Gamma_0 = 800$, while varying the screening parameter $\kappa_0$ as 0.75, 1.0, 1.5, 2.0, 3.0, 5.0, respectively. Note, that for 2D Yukawa systems, when $\Gamma_0 = 800$, the states for $\kappa_0 = 0.75, 1.0, 1.5, 2.0$ are all solids, while the states for $\kappa_0 = 3.0, 5.0$ are liquids [30]. We choose the piston’s moving speed $v_p$ between 0.0007$a_0$ and 2.12$a_0$, so that the corresponding Mach number of this piston moving speed $M_p = v_p/C_i$ is between 0.0786 and 21.3. Following the similar justification in [9], the integration time step is chosen as $dt < 5 \times 10^{-4} a_0/v_p$ to make sure that the equation of motion for dust particles is integrated once as the piston moves less than $0.0005 a_0$. Note, before applying the piston, we integrate $\geq 2 \times 10^5$ steps and double-check the kinetic temperature to make sure that the initial state arrives at the equilibrium of $\Gamma_0$ and $\kappa_0$. In our simulations, we truncate the Yukawa potential at distances where the interparticle potential energy decays to $<10^{-6}$ of the potential energy between two dust particles with the distance of $r = a_0$.

Note that, our simulation method used here does not include the effects of the frictional gas damping rate, which always exists in dusty plasma experiments. In Ref. [9], besides frictionless MD simulations as presented here, Langevin dynamical simulations have also been performed to include the effects of neutral gas damping rate in the dynamics of compressional shocks in dusty plasmas. The comparison between two types of simulations in [9] indicates that, as a dissipation mechanism, the frictional gas damping does modify some behaviors of compressional shocks. In fact, further detailed investigations related to various neutral gas damping rates in compressional shocks are needed, although they are beyond the scope of the current paper.

III. RESULTS AND DISCUSSIONS

A. Shock front speed $D$

To determine the shock front speed $D$, we prepare the spatiotemporal evolution of the number density $n_d$ of dust
shock front. Here the shock front speed is the counted particle number in each bin is just the plotted number of also draw a solid line, whose slope represents the longitudinal sound front speed of slope of this dashed line would just correspond to the shock the postshock regions, marked as the dashed line in Fig. 2. The shock front is the boundary between the preshock and particles, as in Fig. 2. First, since the simulation box has the symmetry in the y direction, we divide the simulation box \((-240 < x < 240\) into 480 narrow rectangular bins with the width of \(a_0\) in the x direction, and then count the particle number in each bin, called the number density \(n_d\). Next, we plot the counted number as the function of the time \(t\) and the location \(x\), as in Fig. 2, which is the spatiotemporal evolution of the number density \(n_d\) of dust particles. Thus, the preshock and postshock regions can be easily distinguished, because of the higher number density in the postshock region from the compressional shocks. Clearly, the piston is at the border between the postshock region and the bottom dark portion, so that the slope of this border would correspond to the piston speed \(v_p\), as marked in Fig. 2.

From this spatiotemporal evolution of number density of Fig. 2, we can easily obtain the shock front speed of \(D\). The shock front is the boundary between the preshock and the postshock regions, marked as the dashed line in Fig. 2. The slope of this dashed line would just correspond to the shock front speed of \(D = 1.64a_0\omega_p\) in Fig. 2. Thus, when the piston moves as a constant speed of \(v_p = 0.707a_0\omega_p\) in the 2D Yukawa system with the condition of \(k_0 = 0.75\) and \(\Gamma_0 = 800\), the generated shock would propagate at a constant speed of \(D = 1.64a_0\omega_p\). When the Yukawa conditions are changed, or the piston moving speed is changed, then the propagation speed of the generated shock would be reasonably modified. For 2D Yukawa systems at the conditions of \(k_0 = 0.75\) and \(\Gamma_0 = 800\), the longitudinal sound speed \(C_l\) is 0.961\(a_0\omega_p\) from the previous study [28], and we draw this speed using the slope of a black solid line in Fig. 2. As a result, the Mach numbers of the piston moving and the shock front speeds are \(M_p = v_p/C_l = 0.735\) and \(M_D = D/C_l = 1.72\), respectively, in Fig. 2.

![FIG. 2. The spatiotemporal evolution of the number density \(n_d\) of particles in our simulation. The simulation box is divided into 480 narrow rectangular bins with the width of \(a_0\) in the x direction, and the counted particle number in each bin is just the plotted number density here. The dashed line corresponds to the propagation of the shock front. Here the shock front speed is \(D = 1.64a_0\omega_p\), while the piston moving speed is \(v_p = 0.707a_0\omega_p\). For comparison, we also draw a solid line, whose slope represents the longitudinal sound speed \(C_l = 0.961a_0\omega_p\) for the 2D Yukawa system with the condition of \(k_0 = 0.75\) and \(\Gamma_0 = 800\) here.](image)

![FIG. 3. The obtained one-particle distribution function \(f_1(\zeta, v_i)\). Here, \(\zeta\) is the Lagrange coordinate \((\zeta = x - Dt)\), so that the simulated box has been divided into three regions of the postshock, shock front, and preshock, as marked in the x axis. To prepare \(f_1(\zeta, v_i)\), we divide the \(\zeta - v_i\) plane into bins in both \(\zeta\) and \(v_i\) axes, and count the particle numbers in each cell (one element of the 2D array \(f_1(\zeta, v_i)\)). The width of the bin in the \(\zeta\) axis is 0.5\(a_0\), while in the \(v_i\) axis, the width is 0.01\(a_0\omega_p\). Then, we divide the counted number in each cell by the total particle number in all cells, to obtain the one-particle distribution function \(f_1(\zeta, v_i)\). For each value of \(\zeta\), we average \(f_1(\zeta, v_i)\) with the corresponding weight, then we obtain the mean particle speed \(\bar{v}_i(\zeta)\), as the solid curve shown here. Note that the piston speed and the initial conditions of the Yukawa system here are the same as those in Fig. 2.](image)

B. Mean particle speed \(\bar{v}\) after shocks

To determine the mean particle speed \(\bar{v}\) after shocks, we compute the one-particle distribution function \(f_1(\zeta, v_i)\) in Fig. 3, as in Ref. [9]. Here, \(\zeta\) refers to the Lagrange coordinate \((\zeta = x - Dt)\), which is the moving coordinate with the generated shocks. To prepare \(f_1(\zeta, v_i)\), first, we count the particle number in different coordinates of \(\zeta\) and \(v_i\). For the spatial axis of \(\zeta\), we divide it into more bins with the width of 0.5\(a_0\). While, for the dynamics axis of \(v_i\), we divide it into bins with the width of 0.01\(a_0\omega_p\), corresponding to \(\approx 1.4\%\) moving speed of the piston at this condition. Then, we divide the counted number in each cell by the total particle number in all cells, to obtain the one-particle distribution function \(f_1(\zeta, v_i)\) in Fig. 3.

Three regions of shocks can be clearly seen in Fig. 3. The postshock, shock front, and preshock regions correspond to the spatial portion of \(\zeta < -20\), \(-20 < \zeta < 5\), and \(\zeta > 5\), respectively. In the shock front of Fig. 3, we can see the wave-like motion, called the dispersive shockwave structure.
that has been observed experimentally in collisionless plasmas [31].

Our obtained one-particle distribution function $f_1(\zeta, v_x)$ of Fig. 3 reflects the kinetic temperature of the studied system. As in Fig. 3, the velocity of the postshock region is mainly distributed around $0.7\bar{a}_0\omega_{pd}$, which means that the particles move much faster along the shock direction. For the preshock region ($\zeta > 5$), the particle distribution satisfies the Maxwell distribution, which has the averaged value of about zero. In Fig. 3, for each location of $\zeta$, the width of the velocity distribution should be directly related to the temperature. Clearly, in Fig. 3, the velocity distribution after shocks is much wider than that in the preshock region, which means that the kinetic temperature of the simulated Yukawa system is much higher after the propagation of shocks.

From the obtained one-particle distribution function $f_1(\zeta, v_x)$, we can calculate the mean particle speed $\bar{v}$ after shocks. First, for each $\zeta$ value, we average the particle moving speed $v_x$ using the weighting factor of $f_1(\zeta, v_x)$, so that we obtain the averaged moving speed of particles $\bar{v}(\zeta)$ in the whole region, as the solid curve shown in Fig. 3. Then, we compute the averaged moving speed of particles $\bar{v}(\zeta)$ only for the postshock region of $\zeta < -20$, to obtain the mean particle speed after shocks $\bar{v}$. In Fig. 3, we find that the mean particle speed after shocks is $\bar{v} = 0.707\bar{a}_0\omega_{pd}$, so that the corresponding Mach number is 0.735, the same as the Mach number of the moving piston $M_p$.

From the obtained one-particle distribution function $f_1(\zeta, v_x)$ in Fig. 3 and that for other simulation conditions, there are three features of the postshock region that can be easily observed. First, for all of our simulation conditions, we find that, within the simulation uncertainty, the mean particle speed after shocks $\bar{v}$ is always the same as the moving speed of the piston. Second, the mean particle speed $\bar{v}$ is always less than the shock front speed $D$. This result is reasonable, since the shock wave propagation speed cannot exceed the speed of the “atoms” of the media to sustain this shock. Third, when the Mach number of piston moving speed $v_p$ is larger, the kinetic temperature of the postshock region is increased too high that the speed of a few particles in the shock front region may exceed the shock front speed $D$ so that they can penetrate into the preshock region, which has also been observed in Ref. [9].

Note that, for the typical motion of one dust particle in the postshock region, the velocity in the $x$ direction far more than that in the $y$ direction, so that we mainly focus on the velocity in the $x$ direction after shocks. Unless particularly stated after this subsection, the velocity $v$ refers to the velocity $v$ in the $x$ direction.

C. Shock Hugoniot curves

As the major result of this paper, we find the $D - \bar{v}$ relationship for the compressional shocks in our simulated 2D Yukawa systems, as shown in Fig. 4. Here, the shock front speed $D$ and the mean particle speed after shocks $\bar{v}$ are obtained using the methods described above. The $D - \bar{v}$ relation is often called as the shock Hugoniot curve [32,33]. In Fig. 4, each data point comes from one run of our shock simulation at one specific condition. Different symbols correspond to different values of $\kappa_0$ from our simulated 2D Yukawa systems.

FIG. 4. The obtained $D - \bar{v}$ relations, i.e., the shock Hugoniot curves, for 2D Yukawa systems. Various symbols come from different values of $\kappa_0$ from our shock simulations. The parabolic expression of $D = C_0 + B\bar{v} + A\bar{v}^2$ is chosen to fit to these symbols, as the curves shown, and the corresponding fitting coefficients are shown in the legend. Clearly, with the decrease of $\kappa_0$, the stronger repulsion between particles would increase the shock front speed, or lift the shock Hugoniot curves. For comparison, the analytical $D - \bar{v}$ relation for the 2D ideal gas is also presented here.

To better understand physics of the postshock for 2D Yukawa system, instead of relying on these discrete data points in Fig. 4, we seek the underlying relations between various physical quantities in the postshock region. Using analytical expressions to describe these physical quantities would be helpful to quantitatively derive and predict other physical quantities related to the shocks, as used in Refs. [8,34,35].

To describe the relationship between the shock front speed $D$ and the mean particle speed after shocks $\bar{v}$ in the postshock region, we choose the parabolic expression of

$$D = C_0 + B\bar{v} + A\bar{v}^2$$  \hspace{1cm} (3)

to fit our simulation results, as the curves shown in Fig. 4. It is clear that the chosen parabolic expression can fit to our simulation results pretty well. The fitting coefficients of $A$, $B$, and $C_0$ in the parabolic expression are also shown in Fig. 4.

From the fitting result shown in Fig. 4, for a specific value of $\bar{v}$, when $\kappa_0$ is smaller, the corresponding shock front speed $D$ is always larger. For a smaller screening parameter $\kappa_0$, the interaction between particles is stronger, thus the stronger repulsion between particles would result in a faster wave speed [8]. That is to say, a lower $\kappa_0$ value would result in an increase of the shock front speeds $D$. Also, when $\kappa_0$ increases, the fitting coefficient of $A$ is smaller, i.e., the $D - u$ relation (shocks Hugoniot curve) tends to be more linear, probably due to the weaker Yukawa interparticle interaction for a larger $\kappa_0$ value.

The fitting coefficient $C_0$ is the intercept of the parabolic expression, which means the shock front propagate speed when the moving piston speed of $\bar{v}$ close to zero. Thus, the
FIG. 5. The obtained pressure of 2D Yukawa systems in the postshock region. From our derivation of Eq. (5), the pressure in the postshock region can be analytically expressed by the ratio of $\frac{P}{P_0}$ (a), or postshock mean particle speed $\bar{v}$ (b), or shock front speed $D$ (c), respectively. For comparison, the pressure of 2D ideal gas condition ($T_0 = 300$ K) after shocks is also presented.

physics significance of $C_0$ should be the sound speed $C_l$ of the undistributed Yukawa system. Here, we verify that, for all these six $\kappa_0$ values, the obtained six values of the coefficient $C_0$ are well consistent with the corresponding longitudinal sound speeds [28].

For comparison, we also plot the $D - \bar{v}$ relation of the 2D ideal gas in Fig. 4. The relevant derivations for the 2D ideal gas are presented in the Appendix. We find that, for the 2D ideal gas, the $D - \bar{v}$ relation can be analytically expressed as $D_{id} = (3\bar{v}_{id} + \sqrt{9\bar{v}_{id}^2 + 16C_l^2})/4$. There is no interaction between the atoms of the 2D ideal gas, so that the momentum can only transfer through the completely elastic collisions between atoms. Thus, the shock front speeds $D$ would be less than those for Yukawa systems, in which the range of the particle interaction is much longer. Note that the $D - \bar{v}$ relation of the 2D ideal gas is more linear, especially for the higher postshock mean particle speed of $\bar{v}_{id}$.

D. Pressure after shocks

As an application of the shock Hugoniot curves, we obtain analytical expressions of the pressure for 2D Yukawa systems after shocks. The Rankine-Hugoniot jump relations [1] for the conservations of mass, momentum, and energy are

$$\rho_0(D - \bar{v}_0) = \rho(D - \bar{v}),$$

$$P - P_0 = \rho_0(D, \bar{v}_0)(\bar{v} - \bar{v}_0),$$

$$\frac{\rho}{P} + \frac{1}{2}(D - \bar{v})^2 = \frac{\rho_0}{P_0} + \frac{1}{2}(D - \bar{v}_0)^2,$$

respectively. Here, $\rho$ is the density, $P$ is the pressure after shocks, and $e = E/m$ is the specific energy [36]. Note that, for our simulation conditions, there is no translational velocity in the preshock region, so that the velocity of $\bar{v}_0$ approximately equals zero. The conservations of mass and momentum in the Rankine-Hugoniot jump relations contain four variables of $P, D, \bar{v}, \rho$, while the shock Hugoniot curves provide the relationship between the shock front speed $D$ and the mean particle speed after shocks $\bar{v}$. Thus, these four variables of $P, D, \bar{v}, \rho$ can be analytically expressed in the three equations of Eqs. (4a) and (4b) and Eq. (3). As a result, in principle, for these four variables, either one can be analytically expressed as a function of only one variable from the other three, as we do for the pressure in the next paragraph.

As another important result, we obtain the pressure of 2D Yukawa systems after shocks, which is analytically expressed as three functions of different variables, $P(\tau), P(\bar{v}), P(D)$, as

$$P - P_0 = \frac{\rho_0}{2\alpha_0^2}\left(1 - B\eta\right)^2 - 2C_0A\eta^2 - (1 - B\eta)\sqrt{(1 - B\eta)^2 - 4C_0A\eta^2},$$

$$P - P_0 = \rho_0(Av^2 + Bv + C_0)v,$$

$$P - P_0 = \rho_0D\left(-B + \sqrt{B^2 - 4A(C_0 - D)}\right)/2A,$$

respectively. Here, $A, B, C_0$ are the obtained coefficients from the shock Hugoniot curves in Eq. (3), and $\eta = 1 - \frac{1}{\bar{v}}$ is negatively correlated with the volume compressibility of $\frac{\bar{v}}{\bar{v}_0}$ after shocks, where $\tau = 1/\rho$ is the specific volume. From Eq. (5), the pressure after shocks can be expressed analytically as various functions containing only one variable, either $\frac{1}{\bar{v}}$, or postshock mean particle speed $\bar{v}$, or shock front speed $D$. Although the pressure after shocks is expressed by three different functions, the corresponding physics process of these three functions is exactly the same. Since either of four variables of $P, D, \bar{v}, \rho$ can be analytically expressed as the function of only one variable from the other three, we can obtain the physical quantities that are difficult to measure from the physical quantities that are easy to measure in shocks, which is a possible application of Eq. (5) in future experiments.

We present the pressure result in Fig. 5 using the obtained expressions in Eq. (5). The variation trend of $P/P_0$ as the variable of $\tau/\tau_0$ is presented in Fig. 5(a), which well agree with the result in Ref. [9]. As the screening parameter $\kappa_0$ increases, the $P/P_0 - \tau/\tau_0$ Hugoniot curve deviates further away from the 2D ideal-gas curve, which looks
From Eq. (6), the energy increase after shocks equals \( \frac{1}{2}(P + P_0)(\tau_0 - \tau) \), which is equivalent to the area of the shadow in Fig. 6(a). In Fig. 6(a), the straight line, which connects the preshock state \((P_0, \tau_0)\) with the postshock state \((P, \tau)\), is called the Rayleigh line [36]. Substituting the analytical expressions of pressure \( P \) after the shocks of Eq. (5a) into the the Hugoniot equation in Eq. (6), the energy in the postshock region can be easily obtained as

\[
e - e_0 = [(1 - B\eta)^2 - 2C_0A\eta^2] - (1 - B\eta)\sqrt{(1 - B\eta)^2 - 4C_0A\eta^2} / 4A^2\eta^2 + \frac{P_0\eta}{\rho_0}.
\]

To illustrate the quantity of Eq. (7), we present the results of the energy in the postshock region in Fig. 6(b) for \( \Gamma_0 = 800 \), with different screening parameters \( \kappa_0 = 0.75, 1.0, 1.5, 2.0, 3.0, 5.0 \). From Fig. 6(b), as the screening parameter of the postshock region \( \kappa_0 \) increases, we can see that the particles in the postshock region have much smaller specific energy \( e \) for the same volume compressibility \( \frac{v_1}{v_0} \). We attribute this decrease trend to the weaker interaction between the particles caused by the stronger shielding effects of the Yukawa potential for larger \( \kappa_0 \) values.

IV. SUMMARY

In summary, we obtain the shock Hugoniot curves of 2D Yukawa systems using MD simulations. From the spatiotemporal evolution of number density, we calculate the shock front speed of \( D \). Then, the mean particle speed after shocks \( \bar{v} \) is obtained from the one-particle distribution function \( f_i(\xi, \nu_s) \). Thus, we obtain the \( D - \bar{v} \) relations of 2D Yukawa systems, which are also called the shock Hugoniot curves.

Combining the Rankine-Hugoniot jump relations with the shock Hugoniot curves, we derive the analytic expressions for the pressure after shocks. Thus, the pressure after shocks can be directly obtained using either the measured values of the ratio of the specific volume \( \frac{v}{v_0} \) or the mean particle speed \( \bar{v} \) after shocks, or the shock front speed \( D \). Substituting the expression of the pressure into the energy Hugoniot equation, the energy after shocks is obtained from the pressure \( P \) and the specific volume \( \tau \).

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APPENDIX: PRESSURE AFTER SHOCKS AND THE SHOCK HUGONIOT CURVE FOR 2D IDEAL GAS

Here, we provide the derivation of the pressure equation of 2D ideal gas after shocks. For the 2D ideal gas, the specific energy \( e = E/m = k_B T/m = p\tau \), so that the Hugoniot equation of Eq. (6) can be rewritten as \( P\tau - P_0\tau_0 = \frac{1}{2}(P + P_0)(\tau_0 - \tau) \),
or the equivalent form of
\[
P \frac{\tau}{P_0 \tau_0} = \frac{1}{3} \left( \frac{P}{P_0} - \frac{\tau}{\tau_0} \right) + 1. \tag{8}
\]

The pressure after shocks can be calculated by the state equation of Eq. (8), which shows in Fig. 5.

We also provide the derivation of the \( D - \bar{v} \) relation (shocks Hugoniot curve) of the 2D ideal gas. The conservation of mass, momentum in Eq. (4) can be written as the form of
\[
\tau = \tau_0 (1 - \bar{v}/D), \tag{9a}
\]
\[
P = \rho_0 D \bar{v} + P_0, \tag{9b}
\]
respectively. Then, we can replace the pressure \( P \) and the specific volume \( \tau \) in Eq. (8) by Eqs. (9a) and (9b). After the simplification of that equation, we obtain the \( D - \bar{v} \) relation of the 2D ideal gas, which is
\[
D = \frac{3\bar{v} + \sqrt{9\bar{v}^2 + 32P_0 \tau_0}}{4} = \frac{3\bar{v} + \sqrt{9\bar{v}^2 + 16C_i^2}}{4}. \tag{10}
\]

Note, for the 2D ideal gas, the adiabatic sound speed is \( C_i = \sqrt{2P_0 \tau_0} \). Thus, when the mean particle speed after shocks \( \bar{v} \) is small, the obtained shock front speed \( D \) would be reduced to the adiabatic sound speed of \( C_i \). We also present the \( D - \bar{v} \) relation of the 2D ideal gas in Fig. 4.